II. *On the Diameter and Magnitude of the* Georgium Sidus; *with a Deſcription of the dark and lucid Diſk and Periphery Micrometers.* *By* William Herſchel, *Eſq. F. R. S.*

Read November 7, 1782.

IT is not only of the greateſt conſequence to the aſtronomer, but alſo gives the higheſt pleaſure to every intelligent perſon, to have a juſt idea of the dimenſions of the ſolar ſyſtem, and the heavenly bodies that belong to it. As far then as they fall within the reach of our inſtruments, they ought carefully to be examined and meaſured by all the various methods we can invent. Almoſt every ſort of micrometer is liable to ſome inconveniences and deceptions: it will, however, often happen, that we may correct the errors of one inſtrument by the oppoſite defects of another. The meaſures of the diameter of the Georgium Sidus, which were delivered in my firſt paper, differ conſiderably from each other. However, if we ſet aſide the three firſt, on a ſuppoſition (as I have hinted before) that every minute object, which is much ſmaller than what we are frequently uſed to ſee, will at firſt ſight appear leſs than it really is; and take a mean of the remaining obſervations, we ſhall have 4″ 36½‴ for the diameter of the planet. On comparing the meaſures then with this mean, we find but two of them that differ ſomewhat more than half a ſecond from it; the reſt are almoſt all within a quarter of a ſecond of that meaſure. This agreement, in the dimenſions of any other planet, would appear very conſiderable; but not being ſatisfied, when I thought it poſſible to obtain much more accurate meaſures, I employed the lamp-micrometer in preference to the former. The firſt time I uſed it upon this occaſion I perceived, that if, inſtead of two lucid points, we could have an intire lucid diſk to reſemble the Planet, the meaſures would certainly be ſtill more compleat. The difficulty of dilating and contracting a figure that ſhould always remain a circle, appeared to me very conſiderable, though nature, with her uſual ſimplicity, holds out to us a pattern in the Iris of the eye, which, ſimple as it appears, is not one of the leaſt admirable of her inimitable works. However, I recollected, that it was not abſolutely requiſite to have every inſenſible degree of magnitude; ſince, by changing the diſtance, I could without much inconvenience make every little intermediate gradation between a ſet of circles of a proper ſize, that might be prepared for the purpoſe. Intending to put this deſign into practice, I contrived the following apparatus.

A large lanthorn, of the conſtruction of thoſe ſmall ones that are uſed with my lamp-micrometer*, muſt have a place for three flames in the middle, which is neceſſary, in order that we may have the quantity of light required, by lighting one, two, or all of them. The grooves, inſtead of braſs ſliding doors, muſt be wide enough to admit a paſte-board, and three or four thickneſſes of paper. I prepared a ſet of circles, cut out in paſte-board, increaſing by tenths of an inch from two inches to five in diameter, and theſe were made to fit into the grooves of the lamp. A good number of pieces, ſome of white, others of light blue paper, of the ſame ſize with the paſte-boards, were alſo cut out, and ſeveral of them oiled, to render them more tranſparent. The oiled papers ſhould be well rubbed, that they may not ſtain the dry papers when placed together. This apparatus being ready, we are to place behind the paſte-board circle, next to the light, one, two, or more, either blue or white, dry or oiled, papers; and by means of one or more flames, to obtain an appearance perfectly reſembling the diſk we would compare it with. It will be found, that more or leſs altitude of the object, and higher or lower powers of the inſtrument, require a different aſſortment of papers and lights, which muſt by no means be neglected: for if any fallacy can be ſuſpected in the uſe of this apparatus, it is in the degree of light we muſt look for it. In a few experiments I tried with theſe lucid diſks, where I placed ſeveral of them together, and illuminated them at once, it was found, that but very little more light will make a circle appear of the ſame ſize with another, which is one, or even two-tenths of an inch leſs in diameter. A well known and ſtriking inſtance of this kind of deception is the moon, juſt before or after the conjunction, where we may ſee how much the luminous part of the diſk projects above the reſt.

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* Phil. Tranſ. vol. LXXII. p. 166.

The method of uſing the artificial diſks is the ſame which has been deſcribed with the lamp-micrometer, of which this apparatus may be called a branch. We are only to obſerve, that the Planet we would meaſure ſhould be cauſed to go either juſt under, or juſt over, the illuminated circle. It may indeed alſo be ſuffered to paſs acroſs it; but in this caſe, the lights will be ſo blended together, that we cannot eaſily form a proper judgement of their magnitudes. By a good ſcrew to the motions of my teleſcope I have been able, at any time, to keep the Planet oppoſite the lucid diſk for five minutes together, and to view them both with the moſt perfect and undiſturbed attention. The apparatus I employed being now ſufficiently explained, ſeveral alterations that were occaſionally introduced will be mentioned in the obſervations and experiments on the Georgium Sidus, as they follow, in the order of time in which they were made.

Obſervations on the Light, Diameter, and Magnitude, of the Georgium Sidus.

Oct. 22, 1781. The Georgium Sidus was perfectly defined with a power of 227; had a fine, bright, ſteady light; of the colour of Jupiter, or approaching to the light of the Moon.

Nov. 28, 1781. I meaſured the diameter of the Georgium Sidus by the lamp-micrometer, and took one meaſure, which I was aſſured was too large; and one, which I was certain was too little; then taking the mean of both, I compared it with the diameter of the ſtar, and found it to agree very well.

Hence $\frac{\text{Image} = 2{,}4 \text{ inches}}{\text{Diſtance} = 431 \text{ inches}} = \text{tang.}\ {,}0055684$; and $\frac{\text{Angle} = 19' \, 8''{,}}{\text{Power} = 227\ 6}$ = the diameter 5″,06. But the evening was foggy, and the ſtar having much aberration, I was induced to try the above method of extreme and mean diameters, ſuggeſted by the method of altitudes, where two equally diſtant extremes give us a true mean.

Nov. 19, 1781. The diameter meaſured 32¼ parts of my micrometer, the wires being outward tangents to the diſk. On ſhutting them gradually by the ſame light, they cloſed at 24; therefore the difference is 8¼ parts, which, according to my ſcale, gives 5″ 2‴ for the diameter. This was taken with 227, and the meaſure ſeemed large enough. Not perfectly pleaſed with my light, which was rather too ſtrong, I repeated the meaſure, and had 33½ parts; then ſhutting the wires gradually, by *this* light they cloſed at 25: the difference, which is 8½ parts, gives 5″ 11‴.

Aug. 29, 1782. 15 h. I ſaw the Georgium Sidus full as well defined with 460, as Jupiter would have been at that altitude with the ſame power.

Sept. 9, 1782. Circumſtances being favourable, I took a meaſure of the diameter of the Georgium Sidus with the power of 460, and ſilk-thread micrometer. After a proper allowance for the zero, I found 4″ 11‴.

Oct. 2, 1782. I had prepared an apparatus of lucid diſks, and meaſured the diameter of the Georgium Sidus with it. Having only white oiled papers, I placed two of them together, and uſed only a ſingle lamp; but could not exactly imitate the light of the Planet. When I firſt ſaw the Sidus and luminous circle together, I was ſtruck with the different colours of their lights; which brought to my recollection γ Andromedæ, ε Bootis, α Herculis, β Cygni, and other coloured ſtars. The Planet unexpectedly appeared blueiſh, while the lucid diſk had a ſtrong tincture of red; but neither of the colours were ſo vivid and ſparkling as thoſe of the juſt mentioned ſtars.

The diſtance of the luminous circle from the eye (which I always meaſure with deal rods) was 588,25 inches. The circle meaſured 2,35 inches. Hence we have the angle $13' 44''$; which, divided by the power 227, gives $3'',63$ for the diameter of the Planet. I ſuſpected ſome little fallacy from the want of a perfect reſemblance in the light and colour of the artificial diſk to the real appearance of the Planet.

Oct. 4, 1782. I meaſured the diameter of the Georgium Sidus again, by an improvement in my apparatus, for I now uſed pale blue papers, both oiled and plain, inſtead of white; by which means I obtained a reſemblance of colours; and by an aſſortment of one oiled and two dry papers with two lamps burning, I effected the ſame degree of light which the Planet had, and both figures were equally well defined. By firſt changing the diſk, and, when I had one which came neareſt, changing my diſtance, I came at a perfect equality between the Planet and diſk. The meaſure was ſeveral times repeated with great precaution. The reſult was $\frac{2,8}{692,6} = ,0040283$; and $\frac{13' 53'',85}{227} = 3'',67$. If any thing be wanting to the perfection of this meaſure, it is perhaps that the Sidus ſhould be in the meridian, in order to have all the advantages of light and diſtinctneſs.

Oct. 10, 1782. The meaſures of the Planet by the lucid diſk micrometer appearing to me very ſmall, I reſolved to aſcertain the power of my teleſcope again moſt ſcrupulouſly, by an actual experiment, without any deduction from other principles. On a moſt convenient and level plain I viewed two ſlips of white paper, and meaſured their images upon a wall. The diſtances were meaſured by deal rods, every repetition whereof was certainly true to half a tenth of an inch; nor did the direction of the meaſure ever deviate, ſo much as two inches, from a ſtraight line.

Diſtance of the object from the eye in inches	7255,5
Diſtance of the eye from the vertex of the ſpeculum	80,2
Diſtance of the vertex of the ſpeculum from the object	7335,7
Diſtance of the eye from the wall	2292,35
Diameter of the largeſt paper	,99125
Diameter of the ſmalleſt	,5075
Image of the largeſt paper on the wall	73,
Image of the ſmalleſt on the ſame	37,8
Angle ſubtended by the large paper at the vertex of the ſpeculum 27″,87	
Angle ſubtended by its image on the wall, at the eye 1° 49′ 26″,4.	
Power of the teleſcope deduced from the large paper	235,6
Angle ſubtended by the ſmall paper at the vertex of the ſpeculum 14″,27.	
Angle ſubtended by its image on the wall, at the eye, 56′ 40″,9,	
Power of the teleſcope deduced from the ſmall paper	238,3
Mean of both experiments, as being equally good	237,
Focal length of the ſpeculum upon thoſe objects	86,1625
Upon Capella	85,2

And 237 diminiſhed in the ratio of 85,2 to 86,1625 gives 234,3 for the power of the inſtrument upon the fixed ſtars.

It appears then, from theſe experiments, that the power of the teleſcope has not been over-rated; and that, therefore, the meaſures of the Georgium Sidus cannot be found too ſmall on that account.

There is one cauſe of inaccuracy or deception in very ſmall meaſures, long ſuſpected, but never yet ſufficiently inveſtigated. That there is a *diſperſion* of the rays of light in their paſſage through the atmoſphere, we may admit from various experiments; if then the quantity of this diſperſion be, in general, regulated by certain diſpoſitions of the air, and other cauſes, it will follow, that a *concentration* may alſo take place: for ſhould the rays of light, at any time, be leſs diſperſed than uſual, they might with as much reaſon be ſaid to be concentrated, as the mercury of a thermometer is ſaid to be contracted by cold, when it falls below the zero.

Oct. 12, 1782. The night was ſo fine, that I ſaw the Georgium Sidus very plainly with my naked eye. I took a meaſure of its diameter by the lucid diſk, and found, that I was obliged to come nearer, as the Planet roſe higher, and gained more diſtinct light. At the altitude of 52° it was as follows:

$$\frac{3{,}415}{731{,}3} = {,}0046698\text{; and } \frac{16'3'',2}{227} = 4'',24.$$

Oct. 13, 1782. 16h. I viewed the Georgium Sidus with ſeveral powers. With 227 it was beautiful. Still better with 278. With 460, after looking ſome time, very diſtinct. I perceived no flattening of the polar regions, to denote a diurnal motion; though, I believe, if it had had as much as Jupiter, I ſhould have ſeen it. With 625 pretty well defined.

Oct. 19, 1782. The inconvenience ariſing from the quantity of light contained in the lucid diſk, ſuggeſted to me the idea of taking only an illuminated periphery, inſtead of the area of a circle. By this means I hoped to ſee the circle well defined, and yet have but little light to interfere with the appearance of the Planet. The breadth of my lucid periphery was one-twentieth of an inch. The reſult of this meaſure proved $\frac{3,3}{765,45} = ,0041486$; and $\frac{14' \, 15'',69}{227} = 3''77$.

Oct. 26, 1782. In my laſt experiment I found the lucid periphery much broader than I could have wiſhed; therefore, I prepared one of no more than one-fortieth part of an inch in breadth, the outer circle meaſuring very exactly 4,00, and the inner circle 3,95. With this ſlender ring of light illuminated with only one ſingle lamp, I meaſured the Georgium Sidus, by removing the teleſcope to various diſtances; and found at laſt the following reſult: $\frac{4,}{1033,05} = ,0038720$; and $\frac{13' \, 18'',6}{227} = 3'',51$.

Nov. 4, 1782. I was now fully convinced that light, be it in the form of a lucid circle, or illuminated periphery, would always occaſion the meaſures to be leſs than they ſhould be, on account of its vivid impreſſion upon the eye, whereby the magnitude of the object, to which the Planet was compared, would be increaſed. It occurred to me then, that if a lucid circle encroached upon the ſurrounding darker parts, a lucid ſquare border, round a dark circle, would in its turn advance upon the artificial diſk. In my laſt meaſures, where the Planet had been compared to a lucid ring, I had plainly obſerved that the Sidus, which was but juſt equal to the illuminated periphery, was conſiderably larger than the black area contained within the ring. This ſeemed to point out a method to diſcover the quantity of the deception ariſing from the illumination; and conſequently, to furniſh us with a correction applicable to ſuch meaſures;

ſures; which would be *plus*, when taken with a lucid diſk or ring; and *minus*, when obtained from a dark ring or circle. Having ſuſpended a row of paſte-board circles againſt an illuminated ſheet of oiled paper, I cauſed the Georgium Sidus to paſs by them ſeveral times, and ſelected from their number that to which the Planet bore the greateſt reſemblance in magnitude. I produced a perfect equality by ſome ſmall alteration of my diſtance, and the reſult was as follows:

$$\frac{3{,}165}{633{,}95} = {,}0049925 : \text{ hence } \frac{17'\ 9'',8}{227} = 4'',53.$$

I was deſirous of ſeeing what would be the effect of leſſening the light of the illuminated frame, againſt which the dark diſks were ſuſpended, and alſo waited a ſhort time that the Planet might riſe up higher. The meaſure being then repeated at a different diſtance, and with a different black diſk, I obtained the following particulars:

$$\frac{3{,}59}{803{,}05} = {,}0044704 ; \text{ and } \frac{15'\ 22'',1}{227} = 4'',06.$$

I intend to purſue theſe experiments ſtill farther, eſpecially in the time of the Planet's oppoſition, and am therefore unwilling as yet to draw a final concluſion from the ſeveral meaſures. In a ſubject of ſuch delicacy we cannot have too many facts to regulate our judgement. Thus much, however, we may in general ſurmiſe, that the diameter of the Georgium Sidus cannot well be much leſs, nor perhaps much larger, than about four ſeconds. From this, if we will anticipate more exact calculations hereafter to be made, we may gather that the real diameter of that planet muſt be between four and five times that of the earth: for by the calculations of M. DE LA LANDE, contained in a letter he has favoured me with, the diſtance of the Georgium Sidus is ſtated at 18,913, that of the earth being 1. And if we take the latter to be ſeen, at the ſun, under an angle of $17''$, it would ſubtend no more than $,''898$, when removed to the orbit of the Georgium Sidus. Hence we obtain $\frac{4}{,898} = 4,454$; which number expreſſes how much the real diameter of the Georgium Sidus exceeds that of the earth.